IONIZATION OF A NEUTRAL MEDIUM BY AN ELECTRON BEAM

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The secondary-electron energy distribution function in the neighborhood of a narrow electron beam is calculated. The total energy range is subdivided into three regions: the Coulomb region, a region of linear ionization cross sections, and an energy region below the ionization potential. Approximate expressions are found for the secondary-electron density and the plasma frequency in the region of the beam.

When an electron beam passes through a neutral medium, excitation of the medium in a certain region of space takes place along with deceleration and scattering of the electrons (see [1-4]). The ionization of neutral atoms leading to the appearance of secondary electrons is of great importance. Some of them (the sufficiently energetic ones) are capable of producing new ionization events. Because of the cascade process, a plasma region is formed around the beam. In the stationary case, the density of free charges in this region depends on experimental conditions and on the processes that occur.

Theoretical studies of the energy structure of electron–electron collision cascades in matter are well known [5]. These calculations usually do not encompass the entire energy range from zero to some maximum value and do not consider spatial characteristics.

The process under discussion is associated with the possibilities of stimulating breakdown processes in the neighborhood of the electron beam, with excitation of electromagnetic oscillations in the plasma region, with the generation of quasiparticles, etc.

Let an infinitely narrow electron beam pass through a medium with an atom density N. We assume cylindrical symmetry for the problem, neglecting variation of characteristics along the beam, and consider all collisions to be pair collisions. The energy of the incident electrons is ε_0, their velocity is v_0 (we consider them nonrelativistic), and n_0 is the number of electrons per unit length of the beam.

The characteristics of interactions between electrons and atoms of the medium depend strongly on the electron energy. The cross section for the most important process – generation of new electrons – is of a complex nature. As is well known [6], the ionization cross section increases linearly at energies near the ionization potential, having a value of zero at $\varepsilon = \Delta$ (Δ is the ionization energy). When ε reaches a value of $m\Delta$ where $m \approx 2$-7, the ionization cross section passes through a maximum and then decreases monotonically. The position of the maximum corresponds to $\varepsilon \approx 5\Delta$ for the molecular gases N_2 and O_2. If $\varepsilon \gg m\Delta$, the dependence of the ionization cross section and the distribution of energy transfer to ejected electrons approximates the Rutherford law. Therefore, in considering cascade multiplication of electrons, it is convenient to subdivide the entire energy range into three regions. The first region (Coulomb region) covers the broad region from $m\Delta$ to the energy ε_0 of the electrons in the beam. The second region (region of linear cross sections) corresponds to the condition $\Delta \le \varepsilon \le m\Delta$. In the third region, $\varepsilon < \Delta$. New electrons do not appear in this last energy region. The main processes there are deceleration of electrons and their recombination with ions. The difficulties of a theoretical description of this region are associated with the complex nature of electron deceleration at low energies and with the difficulties involved in a description of the kinetics of the establishment of equilibrium between the electron gas and molecules. It is also known [6] that the coefficient of recombination depends strongly on energy. For this region, it is convenient to assume a simple, phenomenological computational scheme. As will be seen from the following, the most

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important characteristics of the cascade region, from the applied point of view, depend relatively little on the value of the recombination coefficient.

Since the electron range decreases as ε decreases, one can assume the dimensions of the perturbed region are determined by the range of electrons with the highest energy (a calculation has been made [7] which confirms this assumption for the nonstationary case). One can assume electrons in the second and third regions migrate slightly and their spatial distribution is determined only by the spatial dispersion of the arrival of electrons of higher energies. In the first region, the electrons move practically linearly along the normal to the beam.

The kinetic equations for the regions are

$$\frac{\partial \varphi_1(r,\varepsilon)}{\partial r} + \frac{1}{r}\varphi_1(r,\varepsilon) = \int_{\varepsilon+\Delta}^{\varepsilon_0} \varphi_1(r,\varepsilon')\frac{b}{\varepsilon'}\frac{d\varepsilon'}{(\varepsilon+\Delta)^2} + \int_{\varepsilon+g}^{\varepsilon_0} \varphi_1(r,\varepsilon')\frac{b}{\varepsilon'}\frac{d\varepsilon'}{(\varepsilon'-\varepsilon)^2} - \frac{b}{\varepsilon g}\varphi_1(r,\varepsilon) \tag{1}$$

$$a\int_{\varepsilon+\Delta}^{m\Delta} \varphi_2(r,\varepsilon')\,d\varepsilon' + a\int_{\varepsilon}^{m\Delta} \varphi_2(r,\varepsilon')\,d\varepsilon' - a\varepsilon\varphi_2(r,\varepsilon) + \int_{\varepsilon+\Delta}^{\varepsilon_0} \varphi_1(r,\varepsilon')\frac{b}{\varepsilon'}\frac{d\varepsilon'}{(\varepsilon+\Delta)^2} + \int_{\varepsilon+g}^{\varepsilon_0} \varphi_1(r,\varepsilon')\frac{b}{\varepsilon'}\frac{d\varepsilon'}{(\varepsilon'-\varepsilon)^2} = 0 \tag{2}$$

$$\alpha n_3^2(r) = \int_{m\Delta}^{\varepsilon_0} \varphi_1(r,\varepsilon')\frac{b}{\varepsilon'}d\varepsilon'\int_0^{\Delta}\frac{d\varepsilon}{(\varepsilon+\Delta)^2} + 2a\Delta\int_{\Delta}^{m\Delta}\varphi_2(r,\varepsilon')\,d\varepsilon' \tag{3}$$

$$(\varphi_k = vf_k(r,\varepsilon))$$

Here, f_k is the electron energy distribution function in the k-th region; r is a radial coordinate measured from the initiating beam; a and b are constants depending on the density of the material; g is the minimum energy transfer in a Coulomb collision; n_k is the electron density in the k-th region, i.e., the result of integration of the distribution function over the energy region; α is the recombination coefficient.

The boundary condition for Eq. (1) has the form

$$\lim_{r\to 0} 2\pi r\,\varphi(r,\varepsilon) = cv_0\varepsilon_0^{-1}(\varepsilon+\Delta)^{-2} \tag{4}$$

where c is a constant proportional to the linear density n_0 of the electrons in the beam and to the density N. One can assume

$$c = \pi\beta^2 n_0 N, \quad \beta = Ze^2$$

where Z is the average charge of the targets.

The constant b is such that

$$b = \pi\beta^2 N$$

An explicit expression for the constant a can be obtained from the condition for the equality of the values of the ionization cross section at the energy $m\Delta$ calculated from the relationships in the first and second regions. This gives

$$a = \pi\beta^2 N / m^2\Delta^2 g$$

Electrons in the energy ranges of the second and third regions produced by the initiating beam are not taken into account in Eqs. (1)-(3). They should be considered separately.

We write

$$a\int_{\varepsilon+\Delta}^{m\Delta} \varphi_2'(\varepsilon')\,d\varepsilon' + a\int_{\varepsilon}^{m\Delta} \varphi_2'(\varepsilon')\,d\varepsilon' - a\varepsilon\varphi_2'(\varepsilon) + cv_0/\varepsilon_0(\varepsilon+\Delta)^2 r_0^2 = 0 \tag{5}$$

$$\alpha n_3'^2 = 2a\Delta\int_{\Delta}^{m\Delta}\varphi_2'(\varepsilon')\,d\varepsilon' + cv_0/2\varepsilon_0 r_0^2\Delta \tag{6}$$

In Eqs. (5) and (6), r_0 is a quantity of the order of $(\sigma N)^{-1}$, where σ is the scattering cross section for electrons having an energy $\sim m\Delta$, i.e., the most mobile electrons in the two energy ranges under discussion.

The analysis may be performed analytically by using a number of approximate operations.

The first term on the right of the equality sign in Eq. (1) is small in comparison with the second (the minimum value of the expression $(\varepsilon + \Delta)^{-2}$ is $[(m+1)\Delta]^{-2}$ while the corresponding factor in the second term is g^{-2} where $g \ll \Delta$). One can omit this term by neglecting those ionization events in the first region which occur with large energy transfer to ejected electrons. The number of such collisions in Coulomb processes is small.

The main contribution to the value of the integral (second term) is given by values ε' close to ε. This makes it possible to write

$$\varphi(r, \varepsilon') = \varphi(r, \varepsilon) + (\varepsilon' - \varepsilon)\, \partial\varphi(r, \varepsilon) / \partial\varepsilon$$

If we substitute this relation in Eq. (1) and introduce the variable $y = r\varphi$, we obtain

$$\frac{\partial y}{\partial r} \approx -\frac{b}{\varepsilon^2}\eta y + \frac{b}{\varepsilon}\eta \frac{\partial y}{\partial \varepsilon} \tag{7}$$

Here, $\eta \approx 2$–4. Equation (7) is a first-order partial differential equation. An exact solution satisfying the condition (4) can be found.

We obtain for the function φ_1

$$\varphi_1(r, \varepsilon) = \frac{cv_0}{2\pi\varepsilon_0}\frac{\varepsilon}{r}(2b\eta r + \varepsilon^2)^{-1/2}[(2b\eta r + \varepsilon^2)^{1/2} + \Delta]^{-2} \tag{8}$$

It is clear from Eq. (8) that the density of electrons in the first group falls more rapidly than r^{-1} as a function of distance from the beam. The average energy of the electrons in the region is greater for larger r, which is explained by increased "eating up" of low-energy electrons. The spatial variation of the distribution function for an arbitrary value of ε has the following nature.

In the region of small r, the decrease in f_1 differs little from the r^{-1} law. At more significant values of r, the rate of fall increases and, at "large" r, approximates an $r^{-7/2}$ law. In other words, the boundary of a spatial region where there is a marked amount of electrons of a given energy is rather sharp.

The regions of large and small distances r are determined respectively by the conditions

$$2b\eta r \gg \varepsilon^2, \qquad 2b\eta r \ll \varepsilon^2$$

When the beam intensity n_0 varies, the distribution functions for all values of r and ε change proportionately. A change in the energy ε_0 entails an inversely proportional change in $f_1(r, \varepsilon)$. We point out there is a change in f_1 when there is a change in N, i.e., in the density of the medium. When $r \to 0$, f_1 increases in proportion to the increase in N. The spatial region of perturbation is narrowed approximately proportionately to N^{-1}.

Knowing the form of the function $\varphi_1(r, \varepsilon)$, Eq. (2) can be investigated. If Eq. (2) is differentiated with respect to ε, then considering that

$$\varphi_2(r, \varepsilon) = \varphi_2(\varepsilon) + \delta(\varepsilon)\, \partial\varphi_2(r, \varepsilon) / \partial\varepsilon$$

we obtain in approximate fashion

$$\frac{\partial\varphi_2}{\partial\varepsilon} + \frac{3}{\varepsilon + \delta_1}\varphi_2 = \frac{S_1}{a(\varepsilon + \delta_1)} \tag{9}$$

$$(\delta \lesssim \Delta, \quad \delta_1 \approx {}^1/_3\Delta)$$

The solutions of the linear equation (9) take the forms

$$\varphi_2(r, \varepsilon) = \frac{1}{a(\varepsilon + \delta_1)^3}\int_{m\Delta}^{\varepsilon}(u + \delta_1)^2 S_1(u, r) + \varphi_2(m\Delta)\frac{(m\Delta + \delta_1)^3}{(\varepsilon + \delta_1)^3} \tag{10}$$

$$S_1(r, \varepsilon) = -\int_{m\Delta}^{\varepsilon_0}\varphi_1(\varepsilon')\frac{2b}{\varepsilon'}\frac{d\varepsilon'}{(\varepsilon + \Delta)^3} + \int_{m\Delta}^{\varepsilon_0}\varphi_1(\varepsilon')_{\varepsilon'}\frac{2b\, d\varepsilon'}{(\varepsilon' - \varepsilon + g)^3} \tag{11}$$

Note that consideration can be limited to the first term in Eq. (11) for S_1 when r is large.

Equation (3) yields

$$n_3(r) = \left\{\frac{1}{\alpha}\left[\frac{b}{2\Delta}\int_{m\Delta}^{\varepsilon_0}\varphi_1(\varepsilon')\frac{d\varepsilon'}{\varepsilon'} + 2a\Delta\int_{\Delta}^{m\Delta}\varphi_2(\varepsilon')\,d\varepsilon'\right]\right\}^{1/2}$$

Thus

$$n(r) = n_1(r) + n_2(r) + n_3(r)$$

The electron densities in the first and second energy ranges, calculated from Eqs. (8), (10), and (11), are

$$n_1(r) = \int_{m\Delta}^{\varepsilon_0}\frac{\varphi_1(r,\varepsilon)}{v}\,d\varepsilon, \qquad n_2(r) = \int_{\Delta}^{m\Delta}\frac{\varphi_2(r,\varepsilon)}{v}\,d\varepsilon$$

The spatial variation of the functions n_2 and n_3 correlate to a considerable extent with the relation $n_1 = n_1(r)$, which is determined by the function $\varphi_1(r, \varepsilon)$. Thus there is interest in the values of n_k in the region closest to the beam, where these quantities are maximum. We turn to Eqs. (5) and (6).

The expression for $\varphi_2'(\varepsilon)$, determined in much the same way as $\varphi_2(\varepsilon)$, is written as

$$\varphi_2'(\varepsilon) \approx \frac{2cv_0}{a\varepsilon_0 r_0^2}\frac{1}{(\varepsilon+\delta_1)^3}\left\{\ln\frac{(m+1)\Delta}{\varepsilon+\Delta} + \frac{1}{2m(m+1)}\right\} \tag{12}$$

Integrating Eq. (12) divided by $(2\varepsilon)^{1/2}M^{-1/2}$, we determine n_2'

$$n_2' \approx \sqrt{2M}\,cv_0 \,/\, 24a\varepsilon_0 r_0^2\Delta^{5/2} \qquad (m \approx 5) \tag{13}$$

For the electron density in the third region for the same values of m, we obtain the approximate expression

$$n_3' \approx \left(\frac{3}{4}\frac{cv_0}{\alpha\varepsilon_0 r_0^2\Delta}\right)^{1/2} \tag{14}$$

Adding Eqs. (13) and (14), we finally write

$$n' \approx \frac{\sqrt{2M}\,cv_0}{24a\varepsilon_0 r_0^2\Delta^{5/2}} + \left(\frac{3}{4}\frac{cv_0}{\alpha\varepsilon_0 r_0^2\Delta}\right)^{1/2}$$

Since $r_0 \sim N^{-1}$, the dependence of n_2' on N is rather strong (n_2' is proportional to N^2). The quantity n_3' is proportional to $N^{3/2}$. For sufficiently large values of the recombination coefficient, the dependence of n' on N is determined by the behavior in the nonrecombination region. At small values of α, n_3' has a dominant influence on the magnitude of n', i.e., the recombination zone dominates.

Similarly, $n' \sim n_0^{1/2}$ at small values of α and n' is a linear function of n_0 with sufficiently intense recombination.

Oscillations may be excited in the plasma surrounding the beam. We write an expression for the plasma frequency ω in the region of maximum density assuming the quantity n_3' makes the main contribution to n':

$$\omega = \left\{\frac{4\pi e^2}{M}\left(\frac{3\pi Z^2 e^4 n_0 \sigma^2 N^3 v_0}{4\alpha\varepsilon_0\Delta}\right)^{1/2}\right\}^{1/2}$$

It is clear that ω depends slightly on the quantities n_0, v_0, and α. We evaluate ω for the case ε_0 = 10 keV, Δ = 15 eV, N = 10^{17} cm^{-3}; $n_0v_0 = 6 \cdot 10^{17}$ sec^{-1}. The last quantity corresponds to an electron current of 100 mA. Since electron–electron collisions are being considered, we replace Z by one. We take the quantities σ and α from experimental data. We assume $\sigma = 10^{-16}$ cm^2, $\alpha = 10^{-12}$ cm$^3 \cdot$ sec^{-1}. The calculation yields

$$n' \approx 1.5\cdot 10^{15}\ \text{cm}^{-3} \qquad \omega \approx 2.2\cdot 10^{12}\ \text{sec}^{-1}$$

This frequency value corresponds to a wavelength less than 1 mm.

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